Marcus Anderson

Homework 6

CS 6515: Introduction to Graduate Algorithms

1.) This problem is within an NP.

For the solution to problem B, we need to verify if the solution set of *k+1* vertices in our induced subgraph is a star. This can be done by checking the adjacency list of graph *G* and confirming that every vertex except one has a size of 1, and only one vertex has a size of *k*. Going through the adjacency list is done in O(n) time, which is also in polynomial time. Thus, problem B is within an NP.

2.) Independent Set - > Star-Search

We can show a reduction from the independent set problem to the star-search problem. Given *G = (V, E)* and *k* for the independent set problem, we first modify graph *G* by adding in a new vertex *v*’ that connects to all other vertices within the graph. This new graph will be *G’* and adding vertex *v’* with edges connecting to every other vertex in the graph takes O(n+ m) time. Next, we pass *G’* and *k,* into the star-search algorithm. If a star is returned (*S)*, we can then confirm if its central vertex with a degree of *k* is the same as our new *v’* vertex. Once confirmed, we transform *S* by removing *v’* and should result in an independent set of vertices. This takes O(n) time to remove *v’* and O(1) time to return NO or the independent set. Finally, this means that there is a star within the induced graph IFF *v’* connects to every vertex in an independent set of at least size *k*.

**References:**

* <https://en.wikipedia.org/wiki/Star_(graph_theory)>

**Collaborators:**

* Lilley, Zachary J: zlilley3@gatech.edu
* Bertrand, James M: [jbertrand9@gatech.edu](mailto:jbertrand9@gatech.edu)
* Ramasamy, Veerajothi: vramasamy9@gatech.edu
* Acker, Joshua R: jacker7@gatech.edu
* Shah, Jeet Hemant: jshah328@gatech.edu